4.5. Deflection and Deflection Angle of the Spline Shaft

The deflection and deflection angle of the Ball Spline shaft need to be calculated using equations that meet the relevant conditions. Tables 1 and 2 (pages B-10 and 11) represent these conditions and the corresponding equations.
Tables 3 and 4 (pages B-13 and 14) show the section moduli (Z) and the geometrical moments of inertia (I) of the spline shaft. Using Z and I values in the tables, the strength and displacement (deflection) of a typical Ball Spline model can be obtained.

Table 1 Deflection and Deflection Angle Equations

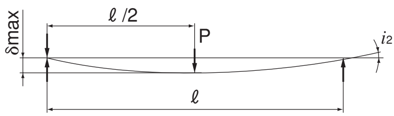
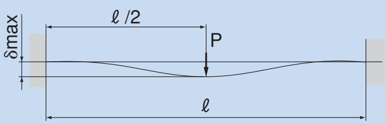
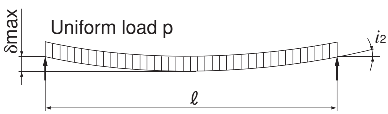
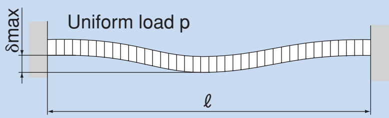
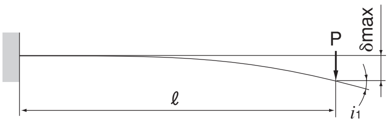
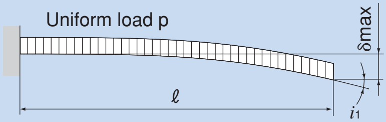
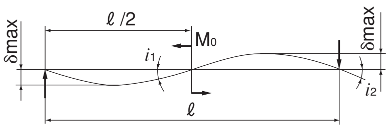
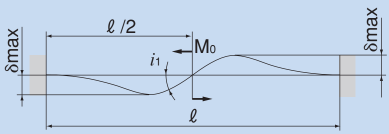
Support method	Service conditions	Deflection equation	Deflection angle equation
Both ends free		$\delta_{max} = \dfrac{P\ell^3}{48EI}$	$i_1 = 0$ $i_2 = \dfrac{P\ell^2}{16EI}$
Both ends fastened		$\delta_{max} = \dfrac{P\ell^3}{192EI}$	$i_1 = 0$ $i_2 = 0$
Both ends free		$\delta_{max} = \dfrac{5p\ell^4}{384EI}$	$i_2 = \dfrac{p\ell^3}{24EI}$
Both ends fastened		$\delta_{max} = \dfrac{p\ell^4}{384EI}$	$i_2 = 0$

Table 2 Deflection and Deflection Angle Equations

Support method	Service conditions	Deflection equation	Deflection angle equation
One end free		$\delta_{max} = \frac{P\ell^3}{3EI}$	$i_1 = \frac{P\ell^2}{2EI}$ $i_2 = 0$
One end fastened		$\delta_{max} = \frac{P\ell^4}{8EI}$	$i_1 = \frac{P\ell^3}{6EI}$ $i_2 = 0$
Both ends free		$\delta_{max} = \frac{\sqrt{3}M_0\ell^2}{216EI}$	$i_1 = \frac{M_0\ell}{12EI}$ $i_2 = \frac{M_0\ell}{24EI}$
Both ends fastened		$\delta_{max} = \frac{M_0\ell^2}{216EI}$	$i_1 = \frac{M_0\ell}{16EI}$ $i_2 = 0$

δ_{max} : Maximum deflection (mm)
i_1 : Deflection angle at loading point
i_2 : Deflection angle at supporting point
M_0 : Moment (N·mm)
P : Concentrated load (N)
p : Uniform load (N/mm)
ℓ : Span (mm)
I : Geometrical moment of inertia (mm^4)
E : Modulus of longitudinal elasticity 2.06×10^5 (N/mm^2)