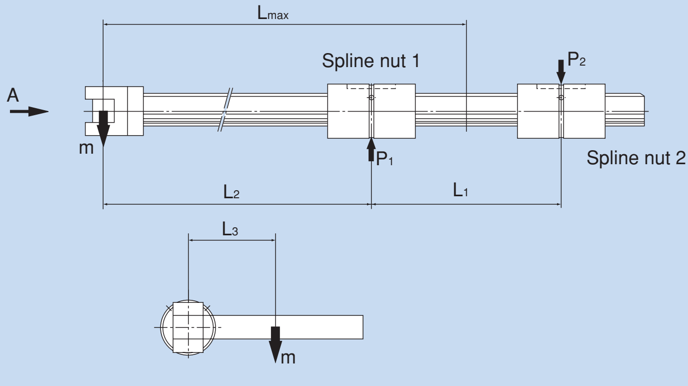
5.5. Example of Calculating the Service Life

5.5.1. Example of Calculation - 1

An industrial robot arm (horizontal)

[Service conditions]

Mass applied to the arm end	m	=50kg	Arm length at maximum stroke	L_{max} =400mm
Stroke	ℓ_s	=200mm		L_2 =325mm
Spline nut mounting span (estimate)	L_1	=150mm		L_3 =50mm



(The Ball Spline type is LBS in this example.)

Fig. 2

Shaft Strength Calculation

Calculate the bending moment (M) and the torsion moment (T) applied on the shaft.

$M=m\times9.8\times L_{max}$ =196000N·mm

$T=m\times9.8\times L_3$ = 24500N·mm

Since the bending and torsion moments are applied simultaneously, obtain the corresponding bending moment (M_e) and torsion moment (T_e), and then determine the shaft diameter based on the greater value. From equations (3) and (4) on page B-9,

$$M_e = \frac{M+\sqrt{M^2+T^2}}{2} \fallingdotseq 196762.7\text{N}\cdot\text{mm}$$

$$T_e = \sqrt{M^2+T^2} \fallingdotseq 197525.3\text{N}\cdot\text{mm}$$

$$M_e < T_e$$

$$\text{From } \therefore T_e = \tau_a \times Z_P$$

$$Z_P = \frac{T_e}{\tau_a} \fallingdotseq 4031\text{mm}^3$$

Thus, judging from table 3 on page B-13, the nominal shaft diameter that meets Z_p is at least 40 mm.

Average Load P_m

Obtain an applied load value when the arm is extended to the maximum length (P_{max}), and another when the arm is contracted (P_{min}). Based on the values obtained, calculate the average load on the spline shaft nut.

$$P_{1max} = \frac{m \times 9.8\,(L_1+L_2)}{L_1} \fallingdotseq 1551.7\text{N}$$

$$P_{2max} = \frac{m \times 9.8 \times L_2}{L_1} \fallingdotseq 1061.7\text{N}$$

When the arm is contracted

$$P_{1min} = \frac{m \times 9.8 \times [(L_2 - \ell_s) + L_1]}{L_1} \fallingdotseq 898.3\text{N}$$

$$P_{2min} = \frac{m \times 9.8 \times (L_2 - \ell_s)}{L_1} \fallingdotseq 408.3\text{N}$$

As this load is monotonically varying as shown in the diagram on page B-20, calculate the average load using the equation (2) on page B-20.

The average load (P_{1m}) on spline nut 1:

$$P_{1m} \fallingdotseq \frac{1}{3}(P_{1min} + 2P_{1max}) = 1333.9\text{N}$$

The average load (P_{2m}) on spline nut 1:

$$P_{2m} \fallingdotseq \frac{1}{3}(P_{2min} + 2P_{2max}) = 843.9\text{N}$$

Obtain the torque applied on one spline nut.

$$T = \frac{m \times 9.8 \times L_3}{2} = 12250\text{N} \cdot \text{mm}$$

Since the radial load and the torque are simultaneously applied, calculate the equivalent radial load using equation (9) on page B-16.

$$P_{1E} = P_{1m} + \frac{4 \times T}{3 \times dp \times \cos\alpha} = 1911.4\text{N}$$

$$P_{2E} = P_{2m} + \frac{4 \times T}{3 \times dp \times \cos\alpha} = 1421.4\text{N}$$

Rated Life L_n

Based on the rated life equation (8) on page B-15, each rated life is obtained as follows.

Rated life of the spline nut 1: $L_1 = \left(\frac{f_T \times f_C}{f_W} \times \frac{C}{P_{1E}}\right)^3 \times 50 = 36598.9\text{km}$

Rated life of the spline nut 2: $L_2 = \left(\frac{f_T \times f_C}{f_W} \times \frac{C}{P_{2E}}\right)^3 \times 50 = 88996.8\text{km}$

f_T : Temperature factor = 1 (from Fig. 1 on page B-17)
f_C : Contact factor = 0.81 (from table 1 on page B-17)
f_W : Load factor = 1.5 (from table 2 on page B-17)
C : Basic dynamic load rating = 31.9 kN (model LBS40)

Given the rated life obtained for each spline nut above, the rated life of the Ball Spline unit is equal to that of spline nut 1, which is 36,598.9 km.

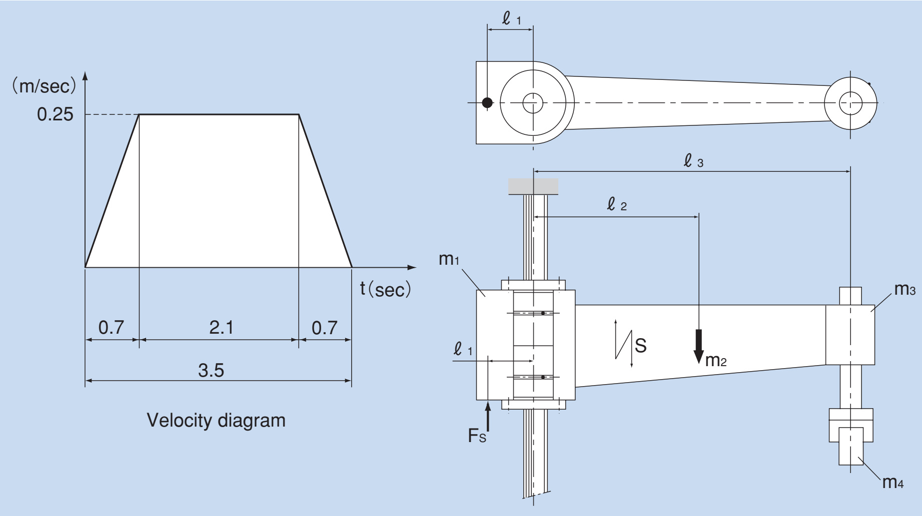
5.5.2. Example of Calculation - 2

[Service conditions]

Thrust position : F_S
Stroke speed : V_{max}=0.25m/sec
Acceleration : a=0.36m/sec^2 (from the respective velocity diagram)
Stroke : S=700mm
Housing mass : m_1=30kg
Arm mass : m_2=20kg
Head mass : m_3=15kg
Workpiece mass : m_4=12kg

Distance from the thrust position to each mass
ℓ_1= 200mm ℓ_2=500mm
ℓ_3=1276mm

Cycle (1 cycle: 30 sec)
1.Descent (3.5sec) 2. Stationary (1sec) : with a workpiece
3.Ascend (3.5sec) 4. Stationary (7sec)
5.Descent (3.5sec) 6. Stationary (1sec) : without a workpiece
7.Ascend (3.5sec) 8. Stationary (7sec)



(The Ball Spline type is LBF in this example.)

Fig. 3

Shaft Strength Calculation

Calculate the shaft strength while assuming the shaft diameter to be 60 mm (with two spline nuts in contact with each other).

Calculating the Moment (M_n) Applying on the Spline Nut during Acceleration, Uniform Motion and Deceleration with Different Masses (m_n)

Applied moment during acceleration : M_1

$$M_1 = m_n \times 9.8 \left(1 \pm \frac{a}{g}\right) \times \ell_n \quad \cdots\cdots\cdots (a)$$

Applied moment during uniform motion : M_2

$$M_2 = m_n \times 9.8 \times \ell_n \quad \cdots\cdots\cdots (b)$$

Applied moment during deceleration : M_3

$$M_3 = m_n \times 9.8 \left(1 \pm \frac{a}{g}\right) \times \ell_n \quad \cdots\cdots\cdots (c)$$

m_n : Mass (kg)

a : Acceleration (m/sec^2)

g : Gravitational acceleration (m/sec^2)

ℓ_n : Offset from each loading point to the trust center (mm)

Assume:

$$A = \left(1 + \frac{a}{g}\right), \quad B = \left(1 - \frac{a}{g}\right)$$

●During descent

From equation (c),

$$M_{m1} = m_1 \times 9.8 \times B \times \ell_1 + m_2 \times 9.8 \times B \times (\ell_1 + \ell_2) + m_3 \times 9.8 \times B \times (\ell_1 + \ell_3)$$
$$= 398105.01 N \cdot mm$$

From equation (b),

$$M_{m2} = m_1 \times 9.8 \times \ell_1 + m_2 \times 9.8 \times (\ell_1 + \ell_2) + m_3 \times 9.8 \times (\ell_1 + \ell_3)$$
$$= 412972 N \cdot mm$$

From equation (a),

$$M_{m3} = m_1 \times 9.8 \times A \times \ell_1 + m_2 \times 9.8 \times A \times (\ell_1 + \ell_2) + m_3 \times 9.8 \times A \times (\ell_1 + \ell_3)$$
$$= 427838.99 N \cdot mm$$

●During ascent

From equation (a),

$M_{m1}' = m_1 \times 9.8 \times A \times \ell_1 + m_2 \times 9.8 \times A \times (\ell_1 + \ell_2) + m_3 \times 9.8 \times A \times (\ell_1 + \ell_3)$

$= 427838.99 N \cdot mm$

From equation (b),

$M_{m2}' = m_1 \times 9.8 \times \ell_1 + m_2 \times 9.8 \times (\ell_1 + \ell_2) + m_3 \times (\ell_1 + \ell_3)$

$= 412972 N \cdot mm$

From equation (c),

$M_{m3}' = m_1 \times 9.8 \times B \times \ell_1 + m_2 \times 9.8 \times B \times (\ell_1 + \ell_2) + m_3 \times 9.8 \times B \times (\ell_1 + \ell_3)$

$= 398105.01 N \cdot mm$

●During descent (with a workpiece loaded)

From equation (c),

$M_{m1}'' = M_{m1} + m_4 \times 9.8 \times B \times (\ell_1 + \ell_3)$

$= 565433.83 N \cdot mm$

From equation (b),

$M_{m2}'' = M_{m2} + m_4 \times 9.8 \times (\ell_1 + \ell_3)$

$= 586549.6 N \cdot mm$

From equation (a),

$M_{m3}'' = M_{m3} + m_4 \times 9.8 \times A \times (\ell_1 + \ell_3)$

$= 607665.37 N \cdot mm$

●During ascent (with a workpiece loaded)

From equation (a),

$M_{m1}''' = M_{m1}' + m_4 \times 9.8 \times A \times (\ell_1 + \ell_3)$

$= 607665.37 N \cdot mm$

From equation (b),

$M_{m2}''' = M_{m2}' + m_4 \times 9.8 \times (\ell_1 + \ell_3)$

$= 586549.6 N \cdot mm$

From equation (c),

$M_{m3}''' = M_{m3}' + m_4 \times 9.8 \times B \times (\ell_1 + \ell_3)$

$= 565433.83 N \cdot mm$

$\therefore M_1 = M_{m1} = M_{m3}' = 398105.01 \quad N \cdot mm$

$M_2 = M_{m2} = M_{m2}' = 412972 \quad N \cdot mm$

$M_3 = M_{m3} = M_{m1}' = 427838.99 \quad N \cdot mm$

$M_1' = M_{m1}'' = M_{m3}''' = 565433.83 \quad N \cdot mm$

$M_2' = M_{m2}'' = M_{m2}''' = 586549.6 \quad N \cdot mm$

$M_3' = M_{m3}'' = M_{m1}''' = 607665.37 \quad N \cdot mm$

Calculating the Equivalent Radial Load Considered to be Applied to the Spline Nut with Different Moments

Relational expression between moment M_n and P_n

$$P_n = M_n \times K \qquad \cdots\cdots\cdots(d)$$

P_n : Equivalent radial load (N)

M_n : Applied moment (N・mm)

K : Equivalent factor (from table 5 on page B-21)

(If two spline nuts of LBF60 contact with each other, K = 0.013)

Calculate the equivalent radial load with different applied moments using equation (d).

$P_{m1} = P_{m3}{'} = M_1 \times 0.013 \fallingdotseq 5175.4N$

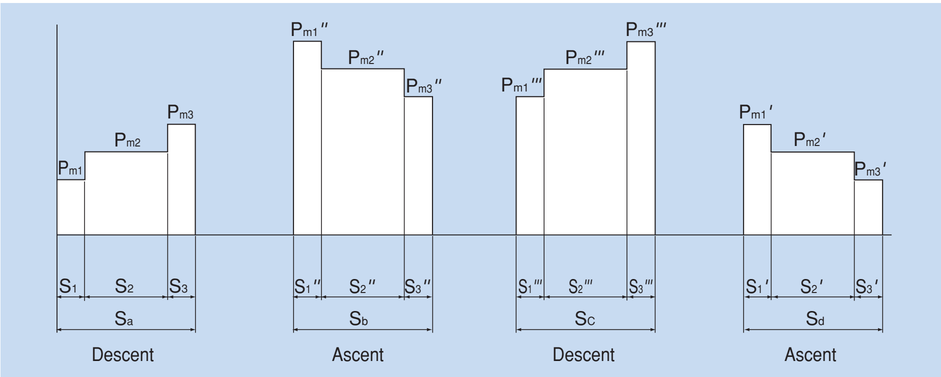
$P_{m2} = P_{m2}{'} = M_2 \times 0.013 \fallingdotseq 5368.6N$

$P_{m3} = P_{m1}{'} = M_3 \times 0.013 \fallingdotseq 5561.9N$

$P_{m1}{''} = P_{m3}{'''} = M_1{'} \times 0.013 \fallingdotseq 7350.7N$

$P_{m2}{''} = P_{m2}{'''} = M_2{'} \times 0.013 \fallingdotseq 7625.2N$

$P_{m3}{''} = P_{m1}{'''} = M_3{'} \times 0.013 \fallingdotseq 7899.7N$



$P_1 = P_{m1} = P_{m3}{'} \fallingdotseq 5175.4N$

$P_2 = P_{m2} = P_{m2}{'} \fallingdotseq 5368.6N$

$P_3 = P_{m3} = P_{m1}{'} \fallingdotseq 5561.9N$

$P_4 = P_{m1}{''} = P_{m3}{'''} \fallingdotseq 7350.7N$

$P_5 = P_{m2}{''} = P_{m2}{'''} \fallingdotseq 7625.2N$

$P_6 = P_{m3}{''} = P_{m1}{'''} \fallingdotseq 7899.7N$

$S = S_a = S_b = S_c = S_d = 700mm$

$S_1 = S_1 = S_1{'} = S_1{''} = S_1{'''} = 87.5mm$

$S_2 = S_2 = S_2{'} = S_2{''} = S_2{'''} = 525mm$

$S_3 = S_3 = S_3{'} = S_3{''} = S_3{'''} = 87.5mm$

Calculating the Average Load P_m

Using equation (1) on page B-19,

$$P_m = \sqrt[3]{\frac{1}{4\times S}\left[2\{(P_1^3\times S_1)+(P_2^3\times S_2)+(P_3^3\times S_3)\}+2\{(P_4^3\times S_3)+(P_5^3\times S_2)+(P_6^3\times S_1)\}\right]}$$

$$\fallingdotseq 6689.5N$$

Calculating the Rated Life L from the Average Load

Using equation (8) on page B-15,

$$L = \left(\frac{f_T \cdot f_C}{f_W} \cdot \frac{C}{P_m}\right)^3 \times 50$$

$$= 7630km$$

f_T : Temperature factor = 1 (from Fig. 1 on page B-17)

f_C : Contact factor = 0.81 (from table 1 on page B-17)

f_W : Load factor = 1.5 (from table 2 on page B-17)

C : Basic dynamic load rating = 66.2 kN (model LBF60)

Given the result above, the rated life of model LBF60 with two spline nuts used in close contact with each other is 7,630 km.